MSO2120 Coursework 1 (2020/21)

Total Marks: 50

Number of questions: 10

Attempt all questions.

Unless told otherwise, Theorems from the course may be used without proof.

Any theorems which are used must be explicitly stated.

You may lose marks for giving a long answer where there is a short one.

You must submit your answers as **one pdf file** on the course **MyLearning page** by:

**23:59** on **Friday** **20th November 2020**.

**Coursework 1**

1. Prove that the following limits hold directly from the definition of a limit:

[2 marks]

[3 marks]

* 1. Prove the following: If a sequence converges to and converges to then the sequence converges to , provided that for all and .

[5 marks]

* 1. Using the result of part a, evaluate the following limit:

ensuring that the steps in your calculations are rigorous and clear.

[3 marks]

* 1. Evaluate the following limit

and prove that the sequence converges to this limit.

[3 marks]

1. Find the upper and lower limits of the following sequences and justify that the values you have found are the upper and lower limits:
   1. ;

[3 marks]

* 1. .

[3 marks]

1. A sequence is defined to be a **Cauchy sequence** if for every there exists such that:

for all

* 1. Prove that if a sequence converges then it is a Cauchy sequence.

[3 marks]

* 1. Is every Cauchy sequence bounded? Justify your claim.

[3 marks]

* 1. Does the following series converge? Prove your claim.

You may use results without proof from the course but you must state them clearly.

[2 marks]

* 1. Does the following series converge? Prove your claim.

You may use results without proof from the course but you must state them clearly.

[2 marks]

1. Consider the following series:
   1. Find the radius of convergence for this series stating any result you use.

[4 marks]

* 1. Determine whether the series converges or diverges at the endpoints of that interval stating clearly any results which you have used.

[4 marks]

1. Let be defined by . Evaluate

and prove your claim directly from the definition of the limit of a function.

[2 marks]

1. A function is said to be **Lipschitz continuous** if there is a constant such that for all .

Show that the function is Lipschitz continuous on[1,2].

[3 marks]

1. Let the polynomial be defined by

for constants .

Prove, stating clearly any results you use from the course, that the function is continuous everywhere.

[2 marks]

1. Prove, stating clearly any results you use from the course, that the function defined by

is continuous everywhere.

[3 marks]