MSO2135 Assessment 2: Problems

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Submission deadline: **Friday** 28th **April at 23:59**.

Learning outcomes:

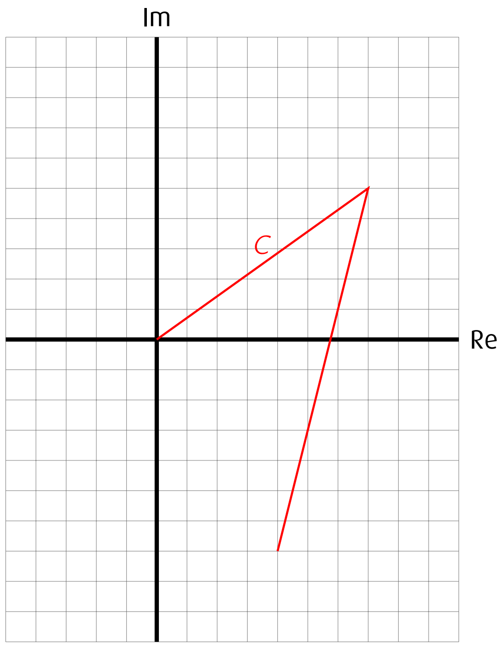
Knowledge 3	Develop reasoned arguments based on fundamental theorems to evaluate complex-valued line integrals.
Skills 4	Use definitions of path integrals to evaluate line integrals of complex functions.
Skills 5	Construct rigorous calculations of improper real-valued integrals by application of the calculus of residues.

Marking

The **Assessment Criteria** can be found on pages 18-20 of the Module Handbook.

Mathematics	40 marks
Presentation	5 marks
Total	**45 marks**

1. Let C be the path consisting of the straight line segment between the origin and the point $7+5i$, and the straight line segment between $7+5i$ and $4-7i$.



   (a) Write down a parametrisation for the path C. 3 marks

   Let $f(z) = 9z + 2$.

   (b) Directly from the definition of complex integration (i.e. without using the Fundamental Theorem of Calculus) evaluate the following integral:

$$\int_C f \; \mathrm{d}z$$

   5 marks

   (c) Write down an anti-derivative of f and verify that the Fundamental Theorem of Calculus holds for the integral in part b). 2 marks

[Total: 10 marks]

2. Consider the function

$$f(z) = \frac{z^2}{(z-3-8i)(z+8-9i)^2} \exp\left(\frac{1}{z-5+8i}\right)$$

(a) Find and classify the singularities of f. What is the largest domain that f is defined on? 4 marks

Consider the paths

$$C = B_2(4+8i) \qquad D = \{ti : 0 \leq t \leq 20\} \cup \{t(-3-2i)+20i : 0 \leq t \leq 10\} \cup \{-t : 0 \leq t \leq 30\}$$
$$E = B_1(-6-8i) \qquad F = \left\{10e^{it} : 0 \leq t \leq \pi\right\} \cup \{t : -10 \leq t \leq 10\}$$

(b) Draw the paths C, D, E and F on an Argand diagram, together with the singularities you found in part (a). 4 marks

(c) Compute the following integrals, justifying your calculations with theory from the course.

$$\int_C f \, \mathrm{d}z \qquad \int_D f \, \mathrm{d}z \qquad \int_E f \, \mathrm{d}z \qquad \int_F f \, \mathrm{d}z$$

12 marks

Hint:

It will be very difficult to perform these integrations directly by parametrising the paths and using the definition of complex integration. Another approach is needed.

[Total: 20 marks]

3. Use the Residue Theorem to evaluate the following real-valued integral:

$$\int_{-\infty}^{\infty} \frac{1}{x^2+2x+2} \, \mathrm{d}x$$

10 marks

[Total: 10 marks]

